Formulas you will be given – know what they mean and how to use them

- $P(A) = \frac{n(A)}{n(S)}$
- $P(A^C) = 1 - P(A)$
- P (A or B) = P(A) + P(B) – P(A and B)
- $P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$
- P (A and B) = P (A|B)* P (B) = P(B) * P(A|B)
- **Disjoint Events:** P (A and B) = 0
- **Independent events:** P (A|B) = P (A)

  P (B|A) = P (B)

  P (A and B) = P (A) * P (B)

1) Find a coin.
   a. Flip the coin five times, what was the probability of seeing a head?

   $P(\text{Head}) = \frac{1}{5} = .2 = 20\%$

   b. Flip the coin ten times, what was the probability of seeing a head?

   $P(\text{Head}) = \frac{6}{10} = .6 = 60\%$

   c. Flip the coin twenty times, what was the probability of seeing a head?

   $P(\text{Head}) = \frac{11}{20} = .55 = 55\%$

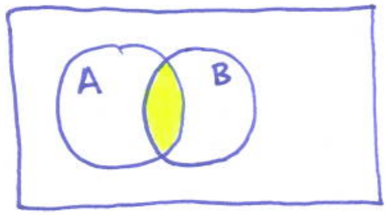
   d. Over time, say we flip a fair coin thousands of times, what do we expect the probability of a heads to be IN THE LONG RUN?

   We expect the probability of seeing a head converge to .5 or 50% as we increase the sample size towards infinity.

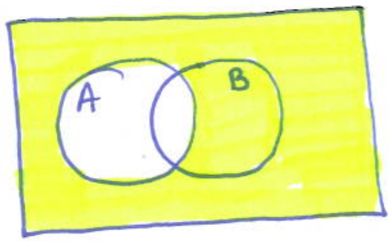
   This is an example of The Law of Large Numbers

2) Draw the Venn Diagrams for the following probabilities – shade the region we're interested in.

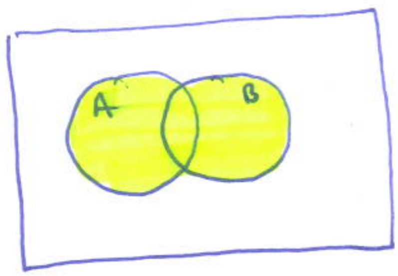
a. P(A and B)



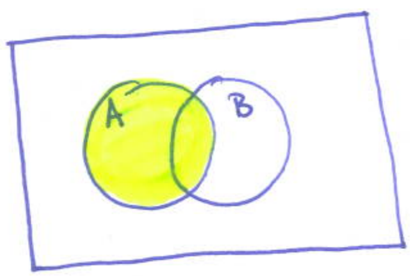
b. P(A^c)



c. P(A or B)



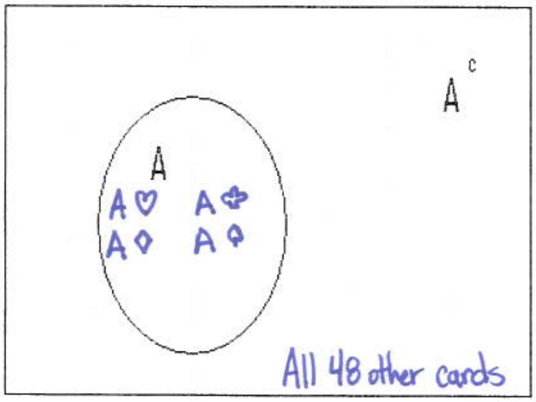
d. P(A)

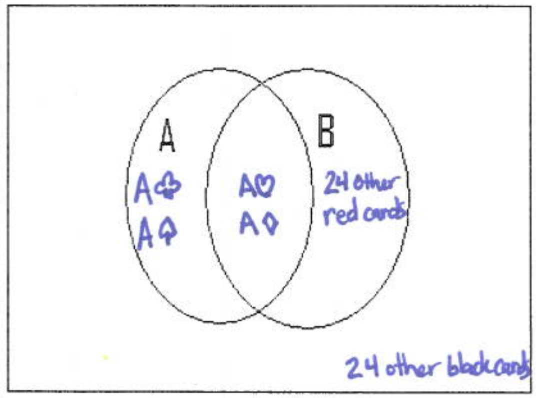


3) Fill out the following diagrams for the events:

A = pulling an ace out of a deck randomly

B = pulling out a red card out of a deck randomly





a. What is the probability of pulling an ace out of a deck randomly, P(A)?

$$P(A) = \frac{\text{\# ways to get an Ace}}{\text{Total \# of cards}} = \frac{4 \text{ Aces}}{52 \text{ total cards}} = \frac{1}{13} \approx .0769 = 7.69\%$$

b. What is the probability of pulling a red card out of a deck randomly, P(B)?

$$P(B) = \frac{\text{\# ways to get a red}}{\text{Total \# of cards}} = \frac{26 \text{ red cards}}{52 \text{ total cards}} = \frac{1}{2} = .5 = 50\%$$

c. What is the probability of pulling a red ace out of a deck randomly, P(A and B)?

$$P(A \text{ and } B) = \frac{\text{\# red aces}}{\text{Total \# of cards}} = \frac{2 \text{ red aces}}{52 \text{ total cards}} = \frac{1}{26} \approx .0385 = 3.85\%$$

d. What is the probability of pulling a red card or an ace out of a deck, P(A or B)?

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = 7.69\% + 50\% - 3.85\% = 53.84\%$$

Fractions are better $\longrightarrow = \frac{1}{13} + \frac{1}{2} - \frac{1}{26} = \frac{7}{13} \approx .5385 = 53.85\%$

e. What is the probability of pulling an ace out of the deck given that you pulled a red card, P(A|B)?

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{3.85\%}{50\%} = .77 = 7.7\%$$

Fractions are better $\longrightarrow = \frac{1/26}{1/2} = \frac{1}{13} \approx .0769 = 7.69\%$

f. Are A and B independent events?

① $P(A|B) = 7.69\% = P(A)$ **NOTE:** Fractions saved us! ✓

② $P(A \text{ and } B) = 3.85\%$ $P(A) \cdot P(B) = \frac{1}{13} \cdot \frac{1}{2} = \frac{1}{26} = 3.85$ ✓

③ $P(B|A) = \frac{P(A \text{ and } B)}{P(B)} = \frac{1/26}{1/2} = \frac{1}{13} = 7.69\% = P(B)$ ✓

Note that we only need to check one as any one implies the other two work & the events are independent.

g. Are A and B mutually exclusive? If not, name a mutually exclusive event for each.

No, they are not mutually exclusive. P(A and B) is not zero so we know the events can happen together.

4) A woman, Sally Clark, was found guilty of the murder of her two sons. The chance of a family suffering sudden infant death syndrome was 1 in 8,500. Pediatrician Professor Roy Meadow, testified that the chance of two children from an affluent family suffering from sudden infant death syndrome was 1 in 73 million by calculating $\left(\frac{1}{8500}\right) * \left(\frac{1}{8500}\right)$. The judgment was that it was far too unlikely that the deaths were caused by the disease and must have been inflicted otherwise. Do you agree with the verdict, why or why not? If you don't agree explain why and state what additional information you would need.

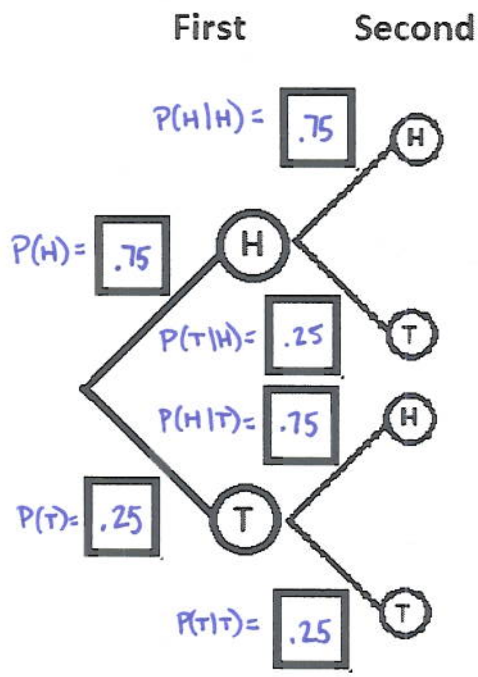
I do not agree w/ the verdict because this calculation assumes that the deaths were independent by using $P(A \text{ and } B) = P(A)P(B)$ and we know that many diseases are genetic. What we would need

① To check the independence assumption

② Find $P(B|A)$ and calculate $P(A \text{ and } B) = P(A)P(B|A)$ if independence doesn't check out.

A CONI research paper showed empirical evidence that suggested $\frac{89}{1000} = 8.9\%$ was the probability of a second sudden death given there was a first. Thus $P(A \text{ and } B) = \frac{1}{8500} * \frac{89}{1000} = \frac{89}{8500000} = .00001047$ or about one in 100,000 which is much more likely than 1 in 73 million.

5) Fill out the following tree diagram for flipping an unfair coin, a coin that lands on heads three quarters of the time, and use to answer the following questions.



Note: the tosses are independent

e. Find the probability of the coin landing on heads for the first toss.

$P(\text{Heads}_1) = .75$

f. Find the probability of the coin landing on heads for the second toss.

$P(\text{Heads}_2) = .75$

g. Find the probability that two heads are flipped in a row.

$P(\text{Heads}_1 \text{ and Heads}_2) \underset{\uparrow}{=} P(\text{Heads}_1) \times P(\text{Heads}_2) = .75 \times .75 = .5625$

By independence

h. Find the probability that a head and a tail are flipped in the two tosses, regardless of order.

Options

HH ✗
HT ✓
TH ✓
TT ✗

$P(1 \text{ Head and } 1 \text{ Tail}) = P(HT \text{ or } TH) \underset{\uparrow}{=} P(HT) + P(TH) \underset{\uparrow}{=} P(H)P(T) + P(T)P(H) = (.75)(.25) + (.25)(.75)$

By Mutually Exclusive

By independence

$= .375$

i. Is flipping a heads on the first toss and flipping a head on the second toss independent?

① $P(Heads_2 | Heads_1) = .75 = P(Heads_2)$ ✓

② $P(Heads_1 \text{ and } Heads_2) = .75 * .75 = .5625 = P(Heads_1) * P(Heads_2)$ ✓

③ $P(Heads_1 | Heads_2)$ ← this really doesn't make sense here

Yes they are independent.

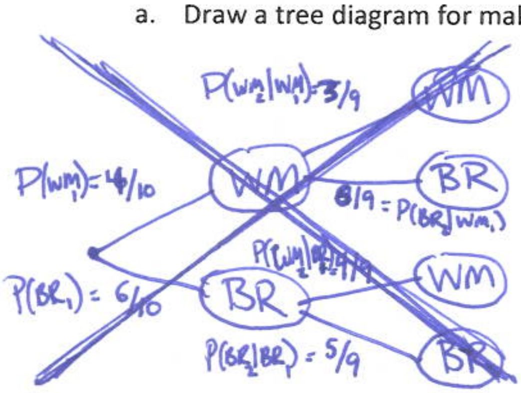
j. What is a mutually exclusive, or disjoint, event for the event flipping a heads on the first trial and why is it so?

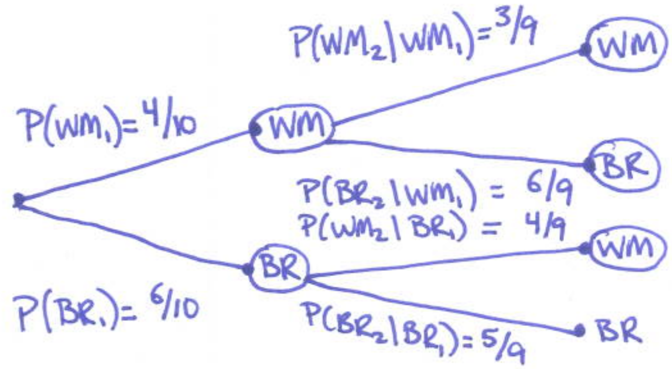
Flipping a tails on the first trial, because they can't happen at the same time.

$P(Heads_1 \text{ and } Tails_1) = 0$.

6) There's a bag of ten Jolly Ranchers somewhere that only has watermelon and blue raspberry jollies in it. There are 6 blue raspberry jollies in the bag, draw a tree diagram for making two selections without replacement.

a. Draw a tree diagram for making two selections without replacement.





This was awful! I'm better than that!

b. Find the probability of choosing a blue raspberry jolly rancher on the first try.

$P(BR_1) = \frac{6}{10} = .6 = 60\%$

c. Find the probability of choosing a blue raspberry jolly rancher on the second try, given that the first choice was watermelon.

$P(BR_2 | WM_1) = \frac{6}{9} = .\overline{66} = 66.\overline{66}\%$

d. Find the probability that two watermelon jollies were chosen in a row.

$P(WM_1 \text{ and } WM_2) = P(WM_1)P(WM_2|WM_1) = \frac{4}{10} * \frac{3}{9} = \frac{12}{90} = \frac{2}{15} \approx .\overline{13} = 13.\overline{33}\%$

e. Find the probability that a watermelon and a blue raspberry jolly were chose, regardless of order.

Options

$WM_1\ WM_2$ X

$WM_1\ BR_2$ ✓

$BR_1\ WM_2$ ✓

$BR_1\ BR_2$ X

$P(1\text{WM and }1\text{BR}) = P(WM_1 BR_2 \text{ or } BR_1 WM_2) \underset{\uparrow}{=} P(WM_1 BR_2) + P(BR_1 WM_2) = \left(\frac{4}{10}\right)\left(\frac{6}{9}\right) + \left(\frac{6}{10}\right)\left(\frac{4}{9}\right)$

By Mutually Exclusive

$= \frac{24}{90} + \frac{24}{90} = \frac{48}{90} = \frac{8}{15}$

$\approx .\overline{53} = 53.\overline{33}\%$

f. Is choosing jolly ranchers from the bag without replacement an example of independent trials? Why or why not?

No because the probabilities change as we remove elements. For example the probability of selecting a blue raspberry first is $\frac{6}{10}$, but after the selection there are only 9 jolly ranchers left 5 of which are blueraspberry $\rightarrow P(BR_2|BR_1) = 5/9 \neq 6/10$.

g. What is a mutually exclusive, or disjoint, event for choosing a blue raspberry jolly rancher on the first try?

Choosing a watermelon on the first try is a mutually exclusive event because it can't happen at the same time,

$P(BR_1 \text{ and } WM_1) = 0$.

7) Consider the following contingency table, based on demographics and drink preference, and answer the following questions.

	Coffee Preference	Other Breakfast Drink	Total
Southern US	910	90	1000
Northern US	780	220	1000
Total	1690	310	2000

a. What's the probability that a randomly selected person prefers coffee to other beverages?

$P(\text{coffee}) = \frac{1690}{2000} = \frac{169}{200} \approx .845 = 84.5\%$

b. What's the probability that a randomly selected person prefers something other than coffee? How many ways can you think to calculate this?

$P(\text{Coffee}^c) = 1 - \frac{169}{200} = 15.5\%$ $\boxed{\text{Complement Rule}}$

$P(\text{Coffee}^c) = \frac{310}{2000} = \frac{310}{200} = .155 = 15.5\%$

c. What's the probability that a randomly selected person prefers coffee, given that they are from the south?

$P(\text{Coffee} \mid \text{South}) = \frac{P(\text{coffee \& South})}{P(\text{South})} = \frac{\frac{910}{2000}}{\frac{1000}{2000}} = \frac{910}{1000} = \frac{91}{100} = .91 = 91\%$

You can think: $= \frac{\text{\# that like coffee from the south}}{\text{Total \# from the south}} = \frac{910}{1000} = 91\%$
of the south as our only sample

d. What's the probability that a randomly selected person from the north, given that they like coffee?

$P(\text{Coffee} \mid \text{North}) = \frac{P(\text{Coffee \& North})}{P(\text{North})} = \frac{\frac{780}{2000}}{\frac{1000}{2000}} = \frac{780}{1000} = \frac{78}{100} = .78 = 78\%$

You can think: $\frac{\text{\# that like coffee from the north}}{\text{Total \# from the north}} = \frac{780}{1000} = 78\%$
of the north
as our only
sample

8) Consider the following contingency table, based on gender and drink preference, and answer the following questions.

	Beer Preference	Other Preference (Not Beer)	Total
Male	82	18	100
Female	56	44	100
Total	138	62	200

a. What's the probability that a randomly selected person prefers beer to other beverages?

$P(\text{Beer}) = \frac{138}{200} = \frac{69}{100} = .69 = 69\%$

b. What's the probability that a randomly selected person prefers something other than beer? What are two ways to go about calculating this?

$P(\text{Beer}^c) = 1 - .69 = .31 = 31\%$

$P(\text{Beer}^c) = \frac{62}{200} = \frac{31}{100} = .31 = 31\%$

c. What's the probability that a randomly selected person prefers beer, given that they are male?

$P(\text{Beer}|\text{Male}) = \frac{P(\text{Beer \& Male})}{P(\text{Male})} = \frac{\frac{82}{200}}{\frac{100}{200}} = \frac{82}{100} = .82 = 82\%$

You can think of males as our only sample : $\frac{\text{\# of males that prefer beer}}{\text{Total \# of males}} = \frac{82}{100} = 82\%$

d. What's the probability that a randomly selected person doesn't prefer beer, given that they are female?

$P(\text{Beer}^c|\text{Female}) = \frac{P(\text{Beer}^c \text{ \& Female})}{P(\text{Female})} = \frac{\frac{44}{200}}{\frac{100}{200}} = \frac{44}{100} = .44 = 44\%$

You can think of females as our only sample : $\frac{\text{\# of females that have an other preference}}{\text{Total \# of females}} = \frac{44}{100} = 44\%$